*Int. J. Heat Mass Transfer.* Vol. 14, pp. 499–502. Pergamon Press 1971. Printed in Great Britain

# THE DEVELOPMENT OF LAMINAR FLOW IN THE ENTRANCE REGION OF A CONCENTRIC ANNULUS WITH A POROUS INNER WALL

W. DAVID MORRIS
School of Applied Sciences, University of Sussex, Falmer, Sussex, England

(*Received* 5 *February* 1970 *and in revised form* 28 *June* 1970)

## NOMENCLATURE

$d$, hydraulic diameter;
$M$, wall mass flow parameter $[Re\,.\,u/w_m]$;
$P$, pressure parameter $\dfrac{p_0 - p}{\rho w_m^2}$;
$p$, pressure;
$p_0$, pressure at entry station;
$r$, radial coordinate;
$R$, non-dimensional radial coordinate $[r/d]$;
$Re$, Reynolds number $[w_m d_\rho/\mu]$
$u$, radial velocity component;
$w$, axial velocity component;
$w_m$, mean axial velocity at the entry station;
$W$, non-dimensional axial velocity $[w/w_m]$;
$z$, axial coordinate;
$Z$, non-dimensional axial coordinate $\dfrac{Z}{d\,Re}$.

Greek symbols
$\rho$, mass density;
$\mu$, viscosity;
$\psi$, stream function;
$\eta$, non-dimensional cross stream variable $\dfrac{\phi - \phi_i}{\phi_0 - \phi_i}$;
$\Phi$, non-dimensional stream function $[\psi/\rho d^2 w_m]$;
$\tau$, shear stress.

Subscripts
$i$, inner wall;
0, outer wall.

## INTRODUCTION

A KNOWLEDGE of the influence of suction or blowing at the walls of ducted flow geometries is important for the reliable design of many engineering components, notably the effusion cooled turbine blade and the combustion chambers of gas turbines.

The present communication presents the results of an investigation into the effect of either suction or blowing at the inner wall of a concentric annular duct on the development of laminar flow.

## ANALYSIS

Consider laminar flow in a concentric annulus with the motion referred to an axisymmetric coordinate frame with axial and radial coordinates $z$ and $r$ respectively. It is assumed that all relevant properties of the fluid are constant and that the level of suction or blowing at the walls permits the elliptic momentum equations to be truncated to parabolic form using the usual boundary-layer approximations.

It can be shown that the basic conservation equations of mass and momentum combine under these circumstances to give

$$\frac{\partial W}{\partial Z} = \frac{1}{W}\frac{\mathrm{d}P}{\mathrm{d}Z} + \frac{\partial}{\partial \phi}\left[WR^2 \frac{\partial W}{\partial \phi}\right] \tag{1}$$

where

$$W = \frac{w}{w_m}, \qquad P = \frac{p_0 - p}{\rho w_m^2}, \qquad R = \frac{r}{d},$$

$$Z = \frac{z}{d\,Re}, \qquad \phi = \frac{\psi}{\rho d^2 w_m}$$

and $\psi$ is the dimensional stream function defined by the relationships

$$\frac{\partial \psi}{\partial r} = \rho r w; \qquad \frac{\partial \psi}{\partial z} = -\rho r u. \tag{2}$$

All symbols are defined in the nomenclature.

The independent variables $Z$ and $\phi$ may be transformed to $Z$ and $\eta$ where, following the proposal of Spalding and Patankar [1, 2] $\eta = \dfrac{\phi - \phi_i}{\phi_0 - \phi_i}$ to give

$$\frac{\partial W}{\partial Z} + \left\{\frac{\eta[M_0R_0 - M_iR_i] + M_iR_i}{[\phi_0 - \phi_i]}\right\}\frac{\partial W}{\partial \eta} - \frac{1}{[\phi_0 - \phi_i]^2}\frac{\partial}{\partial \eta}\left\{WR^2\frac{\partial W}{\partial \eta}\right\} = \frac{1}{W}\frac{\mathrm{d}P}{\mathrm{d}Z} \tag{3}$$

where

$$M_i = Re\,u_i/w_m, \text{ and } M_0 = Re\,u_0/w_m.$$

A solution of equation (3) is sought subject to the restrictions that at the inner and outer walls the axial velocity is zero and the distribution of suction or blowing is specified. Also the axial velocity profile at the entry plane is specified. Equation (3) was solved using a modified form of the numerical procedure proposed by Spalding and Patankar [1, 2]. In order to determine the pressure gradient at a given downstream location a rapid iterative procedure was used which ensured that the resulting velocity profile satisfied continuity to a predetermined level of acceptability. The method is an adaptation of that used by Owen [3] and Bayley and Owen [4] and in principle has also been used by Worsøe-Schmidt and Leppart [5]. To improve the accuracy of the prediction near the duct walls use was made of the "slip velocity" concept of Spalding and Patankar [1, 2] together with an asymptotic solution of equation (3) which simultaneously accounts for pressure gradient and mass transfer in the vicinity of a wall. The asymptotic solution in the vicinity of the wall implies the use of small grid spaces in this region. Accordingly grid spaces expanding outwards in geometric progression from the walls towards the centre of the duct were used. Preliminary calculation indicated that a total of 100 grid spaces (50 expanding from the inner wall to $\eta = 0{\cdot}5$ with a step size ratio of 1·1 and then compressing in a similar fashion to the outer wall) gave acceptable results when compared with known test cases listed below. This grid spacing was subsequently used for the results presented here. Further algebraic details are given by Morris [6].

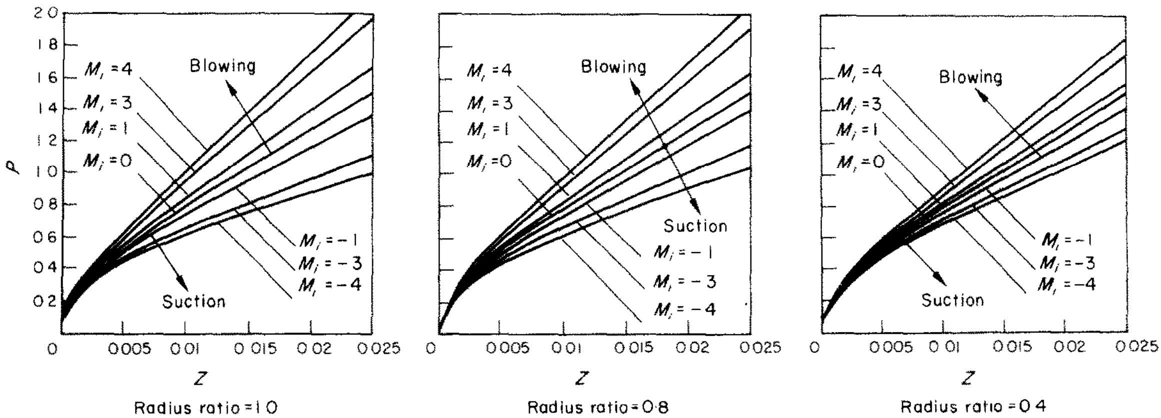



FIG. 1. Influence of uniform suction or blowing at the inner wall of a concentric annulus. (Velocity profile is uniform at entry.)

## RESULTS AND DISCUSSION

To ensure confidence in the procedure the results of three test cases were compared with those of other workers. Specifically these were:

1. Flow development between impervious parallel plates having an initially uniform velocity profile. (See Bodoia and Osterle [7])
2. Flow development along an annular duct with impervious walls and an initially uniform velocity profile. (See Sparrow and Lin [8])
3. Flow development between infinite parallel plates with developed Poiseuille flow at entry and uniform suction over both walls. (See Berman [9]).

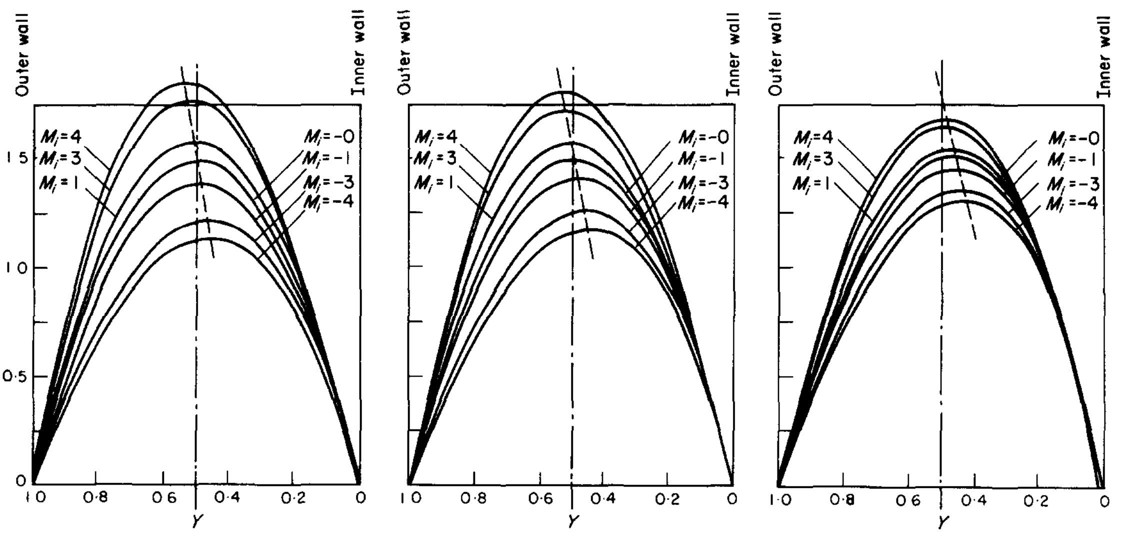
For each of these test cases excellent agreement with the relevant reference was found (see [7–9]). Indeed the present predictions were generally indistinguishable from the graphical data given by the respective authors.

Having satisfactorily demonstrated that the calculation procedure gave meaningful results, the influence of uniform suction or injection at the inner walls of the annulus was investigated for the case where the entry velocity profile was uniform. Similar tendencies were noted for all radius ratio values used for the calculations and some of the salient features of these results will now be presented.

Figure 1 illustrates the influence of either suction or injection of fluid on the pressure parameter, $P$, for radius ratio values of 1·0, 0·8 and 0·4. Compared with the case when both walls are solid, injection of fluid at the inner wall produces a steeper gradient of the pressure parameter with this effect accentuated as the injection parameter, $M_i$, is increased. This behaviour reflects the increased fall off in actual pressure along the duct which is required in order to accelerate the injected fluid in the axial direction. The interaction between this effect and the changes in wall shear stress under different flow conditions determines the actual pressure distribution according to the basic conservation equations.

Conversely, when suction is permitted along the inner wall the opposite effect is evident. Now the gradient of the presure parameter is lower at any location than the value corresponding to the case where both walls are impervious. Thus the actual pressure at a given location is higher than the solid wall case. With either suction or injection the pressure parameter variation along the duct tended to become linear as the flow progressed along the duct.

When a fluid flows in an annulus which has solid walls it is well known that the velocity profile exhibits a maximum value which is physically located nearer the inner rather than the outer wall. Associated with this is the fact that at a given axial location the shear stress at the inner wall is greater than that at the outer wall. The effect of either suction or injection at the inner wall on the axial velocity profile and the location of the maximum value is typified by Fig. 2. Here




(a) Radius ratio = 1·0 (b) Radius ratio = 0·8 (c) Radius ratio = 0·4

FIG. 2. Influence of uniform suction or blowing at the inner wall or a concentric annulus with uniform velocity profile ($Z = 0{\cdot}03$).

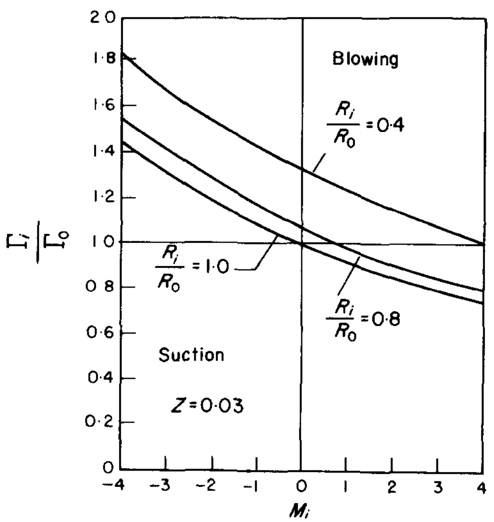



FIG. 3. Typical effect of uniform suction or blowing at the inner wall on the wall shear stress ratio.

the velocity profile at $Z = 0{\cdot}03$ has been drawn for a variety of suction and injection parameters. For all values of the radius ratio injection causes the point of maximum velocity to move towards the outer wall, the higher the value of the injection parameter the greater is the movement of this point towards the outer wall. As expected, the effect of suction is to bring the point of maximum velocity nearer to the inner wall. For the three radius values shown, the locus of the location of maximum velocity is shown as the scored line.

Associated with these distortions of the velocity profile are the variations of the ratio of inner/outer shear stress shown in Fig. 3.

The shear stress ratio, $\Gamma_i/\Gamma_0$ is significantly increased in relation to the case where the walls are solid when suction is permitted at the inner surface. The reverse effect is again apparent when blowing at the inner surface occurs.

## REFERENCES

1. D. B. SPALDING and S. V. PATANKAR, *Heat and Mass Transfer in Boundary Layers*. Morgan-Grampian (1967).

2. D. B. SPALDING and S. V. PATANKAR, A calculation procedure for solving the equations of the two dimensional boundary layer, Imperial College, Mech. Eng. Fept. Rep. TWF/TN/20 (1967).
3. J. M. OWEN, Flow between a rotating and stationary disc, D.Phil. Thesis, University of Sussex (1969).
4. F. J. BAYLEY and J. M. OWEN, Flow between a rotating and stationary disc. *Aeronaut. Q.* Vol. XX (1969).
5. P. M. WORSØE-SCHMIDT and G. LEPPERT, Heat transfer and friction for laminar flow of gas in a circular tube at high heating rate. *Int. J. Heat Mass Transfer* **8**, 1281–1301 (1965).
6. W. D. MORRIS, Developing laminar flow in the entrance region of a concentric annulus with porous inner and outer walls, 70/Me/22 App. Sci. Report, University of Sussex (1970).
7. J. R. BODOIA and J. F. OSTERLE, Finite difference analysis of plane Poiseuille and Couette flow developments, *Appl. Sci. Res.* **10A**, 265–276 (1961).
8. E. M. SPARROW and S. H. LIN, Developing laminar flow and pressure drop in the entrance region of annular ducts. Paper No. 64 FE-1, A.S.M.E. (1964).
9. A. S. BERMAN, Laminar flow in channels with porous walls. *J. Appl. Phys.* **24**, 1232–1235 (1955).

*Int. J. Heat Mass Transfer.* Vol. 14, pp. 502–505. Pergamon Press 1971. Printed in Great Britain

# EFFECT OF REGULARLY SPACED SURFACE RIDGES ON FILM CONDENSATION HEAT TRANSFER COEFFICIENTS FOR CONDENSATION IN THE PRESENCE OF NONCONDENSABLE GAS

**KI. I. CHANG and DONALD L. SPENCER**
University of Iowa, Iowa City, Iowa, U.S.A.

(*Received* 11 *June* 1970)

## NOMENCLATURE

$\bar{h}$, mean heat-transfer coefficient;
$k$, thermal conductivity;
$\mu$, viscosity;
$\rho$, density;
$g$, acceleration of gravity;
$N_{Re}$, Reynolds number.

## INTRODUCTION

A NUMBER of analytical and experimental condensation heat-transfer studies dealing with the influence of condensables have been reported in the literature. For example, Sparrow and Eckert [1], Sparrow and Lin [2] and Minkowycz and Sparrow [3] transformed governing equations in partial differential form into ordinary differential equations by using similarity parameters and solved them numerically. Othmer [4] and Meisenburg *et al.* [5] have showed experimentally the reduction of condensation heat-transfer coefficients for condensing steam in the presence of air.

None of the above-mentioned works, however, give information about possible instability of the flow due to the presence of noncondensables and possible secondary flow associated with the transport of noncondensables.

Spencer *et al.* [6] have reported the existence of instability phenomena, the occurrence of ridge waves and the effect of molecular weight of noncondensables on the condensation heat-transfer process. In this work, Freon 12 was used as a condensable vapor and $N_2$, He and $CO_2$ as noncondensable gases. It was reported that the condensation heat transfer coefficients are dependent on the molecular species of the noncondensable gas and also that a liquid film wave phenomenon exists peculiar to condensation in the presence of noncondensables. These waves were called "ridge waves", since wave crests were vertical rather than horizontal.

They interpreted the occurrence of ridge waves as being evidence that steady state diffusion does not take place under the condition of the experiment at sufficiently high Reynolds number, but rather that cellular motion occurs in which condensable vapor is transported in and noncondensables transported out essentially in eddy motion.

In addition, extremely low heat-transfer coefficients obtained in [6] was interpreted as an evidence of an interfacial resistance effect due to or enhanced by noncondensables.

In order to further clarify the interaction between the condensable vapor and the noncondensables and the formation of ridge waves due to noncondensables, some changes were made to the apparatus used in the above reference work. A $\frac{1}{2}$ in. wide annular brass plate was attached to the